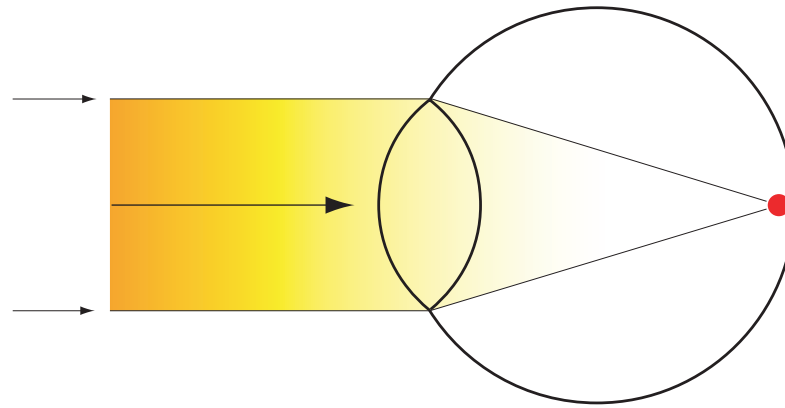


Curriculum Focal Points for Prekindergarten *through* Grade 8 Mathematics

A Quest for Coherence



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

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Preface

As states and local school districts implement more rigorous assessment and accountability systems, teachers often face long lists of mathematics topics or learning expectations to address at each grade level, with many topics repeating from year to year. Lacking clear, consistent priorities and focus, teachers stretch to find the time to present important mathematical topics effectively and in depth.

The National Council of Teachers of Mathematics (NCTM) is responding to this challenge by presenting *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. Building on *Principles and Standards for School Mathematics* (NCTM 2000), this new publication is offered as a starting point in a dialogue on what is important at particular levels of instruction and as an initial step toward a more coherent, focused curriculum in this country.

The writing team for *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* consisted of nine members, with at least one university-level mathematics educator or mathematician and one pre-K–8 classroom practitioner from each of the three grade bands (pre-K–grade 2, grades 3–5, and grades 6–8). The writing team examined curricula from multiple states and countries as well as a wide array of researchers' and experts' writings in creating a set of focal points for pre-K–grade 8 mathematics.

On behalf of the Board of Directors, we thank everyone who helped make this publication possible.

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Introduction

In 1980 the National Council of Teachers of Mathematics (NCTM) published *An Agenda for Action* (NCTM 1980), launching an era of bold professional outreach by describing the shape that school mathematics programs should take. That publication outlined ten recommendations for K–12 mathematics programs, focusing on the fundamental need of students to learn how to solve problems. In 1989, the Council published *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), expanding these recommendations into a vision for mathematics teaching and learning in K–grade 4, grades 5–8, and grades 9–12. *Curriculum and Evaluation Standards* provided major direction for states and school districts in developing their curriculum guidelines. *Principles and Standards for School Mathematics* (NCTM 2000) followed at the turn of the new century, adding underlying principles for school mathematics and clarifying and elaborating on the 1989 Standards for pre-K–grade 2, grades 3–5, grades 6–8, and grades 9–12.

Principles and Standards for School Mathematics remains the comprehensive reference on developing mathematical knowledge across the grades, and the Council continues to produce numerous related publications and services to support, expand, and illuminate this work. *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* extends the Council’s leadership of more than twenty-five years by describing an approach to curriculum development that focuses on areas of emphasis within each grade from prekindergarten through grade 8.

An approach that focuses on a small number of significant mathematical “targets” for each grade level offers a way of thinking about what is important in school mathematics that is different from commonly accepted notions of goals, standards, objectives, or learning expectations. These more conventional structures tend to result in lists of very specific items grouped under general headings. By contrast, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* offers more than headings for long lists, providing instead descriptions of the most significant mathematical concepts and skills at each grade level. Organizing a curriculum around these described focal points, with a clear emphasis on the processes that *Principles and Standards* addresses in the Process Standards—communication, reasoning, representation, connections, and, particularly, problem solving—can provide students with a connected, coherent, ever expanding body of mathematical knowledge and ways of thinking. Such a comprehensive mathematics experience can prepare students for whatever career or professional path they may choose as well as equip them to solve many problems that they will face in the future.

The curriculum focal points presented here offer both immediate and long-term opportunities for improving the teaching and learning of mathematics. They provide ideas that may kindle fruitful discussions among teacher leaders and teachers about areas to emphasize as they consider the developmental needs of their students and examine a year’s program of instruction. Teachers might also see opportunities to develop or select lessons that bring together related topics in meaningful contexts to reinforce or extend the most



important connections, understandings, and skills. The long-term opportunity, however, is for mathematics leaders at every level to use *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* to launch an ongoing, far-reaching, significant discussion with the potential to guide the thinking of the profession in the development of the next generation of curriculum standards, textbooks, and tests. This work may assist in the creation and eventual development of new models for defining curriculum, organizing instruction, developing materials, and creating meaningful assessments that can help students learn critical mathematical skills, processes, and ways of thinking and can measure and communicate what students know about the mathematics that we expect them to learn.

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics thus represents an important, initial step in advancing collaborative discussions about what mathematics students should know and be able to do. Use the focal points presented here to guide discussions as you review, refine, and revise mathematics curricula. Take this opportunity to share the best that we know as we work together to produce improved tools that support our shared goal of a high-quality mathematics education for every student.

1

Why Identify Curriculum Focal Points?

The National Council of Teachers of Mathematics produced *Principles and Standards for School Mathematics* (NCTM 2000) to update and extend the recommendations for learning and teaching mathematics that had appeared in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), *Professional Standards for Teaching Mathematics* (NCTM 1991), and *Assessment Standards for School Mathematics* (NCTM 1995). *Principles and Standards* enunciated the Curriculum Principle, which states, “A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades” (p. 14). Specifically, “a well-articulated curriculum gives teachers guidance regarding important ideas or major themes, which receive special attention at different points in time. It also gives guidance about the depth of study warranted at particular times and when closure is expected for particular skills or concepts” (p. 16).

This definition of curriculum articulation echoes a central question that occupies state and local leaders in mathematics education: *What mathematics should be the focus of instruction and learning at particular grade levels of the pre-K–12 educational system?* As *Principles and Standards* states, “Those who design curriculum frameworks, assessments, instructional materials, and classroom instruction based on *Principles and Standards* will need to make their own decisions about emphasis and order” (p. 31). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* provides one possible response to the question of how to organize curriculum standards within a coherent, focused curriculum, by showing how to build on important mathematical content and connections identified for each grade level, pre-K–8.

Inconsistency in the Placement of Topics by Grade Level in U.S. Mathematics Curricula

Analysis of curricula of countries participating in the Third International Mathematics and Science Study (TIMSS [1997]; now known as the Trends in International Mathematics and Science Study) led to the familiar description of school mathematics in the United States as “a mile wide and an inch deep” (Schmidt, McKnight, and Raizen 1997). In addition, research on the curricular expectations of states and school systems across the country indicates inconsistency in the grade placements of mathematics topics, as well as in how they are defined and what students are expected to learn.

State and local districts, with varying resources for providing leadership in mathematics education, have been working fairly independently to develop student learning expectations, as required by the federal law No Child Left Behind (2002). The result has been a wide variety of mathematics curriculum standards, with little consensus on the placement or emphasis of topics within specific grade levels (Reys et al. 2005). For example, in a study of the mathematics curriculum standards of ten states (Reys et al. 2006), the total number of grade-level expectations in mathematics for grade 4 ranged from 26 to 89 (see table 1).

Table 1. Number of Fourth-Grade Learning Expectations (LEs) per State by Content Strand
(from Reys et al. 2006, p. 20)

	Number & Operations	Geometry	Measurement	Algebra	Data Analysis, Prob & Stat	Total Number of LEs
California	16	11	4	7	5	43
Texas	15	7	3	4	3	32
New York	27	8	10	5	6	56
Florida	31	11	17	10	20	89
Ohio	15	8	6	6	13	48
Michigan	37	5	11	0	3	56
New Jersey	21	10	8	6	11	56
North Carolina	14	3	2	3	4	26
Georgia	23	10	5	3	4	45
Virginia	17	8	11	2	3	41

The Importance of Curricular Focus in Mathematics

Many factors have contributed to the need for a common mathematical focus for each grade level, pre-K–8. These include the increased emphasis on accountability testing, high levels of mobility of both students and teachers, and greater costs of curriculum development. A focused, coherent mathematics curriculum with a national scope has the potential to ease the impact of widely varying learning and assessment expectations on both students and teachers who relocate. In addition, a focused curriculum would allow teachers to commit more time each year to topics receiving special emphasis. At the same time, students would have opportunities to explore these topics in depth, in the context of related content and connected applications, thus developing more robust mathematical understandings.

In a survey of employees of forty-seven educational agencies—those responsible for improving curriculum and instruction in their states—85 percent of the respondents indicated that “national leadership is needed to assist in future articulation of learning expectations in mathematics, particularly from national professional organizations of mathematics teachers (K–12 and university) and mathematicians” (Reys et al. 2005, p. 17). This publication addresses that need.

2

What Are Curriculum Focal Points?

Curriculum focal points are important mathematical topics for each grade level, pre-K–8. These areas of instructional emphasis can serve as organizing structures for curriculum design and instruction at and across grade levels. The topics are central to mathematics: they convey knowledge and skills that are essential to educated citizens, and they provide the foundations for further mathematical learning. Because the focal points are core structures that lay a conceptual foundation, they can serve to organize content, connecting and bringing coherence to multiple concepts and processes taught at and across grade levels. They are indispensable elements in developing problem solving, reasoning, and critical thinking skills, which are important to all mathematics learning.


When instruction focuses on a small number of key areas of emphasis, students gain extended experience with core concepts and skills. Such experience can facilitate deep understanding, mathematical fluency, and an ability to generalize. The decision to organize instruction around focal points assumes that the learning of mathematics is cumulative, with work in the later grades building on and deepening what students have learned in the earlier grades, without repetitious and inefficient reteaching. A curriculum built on focal points also has the potential to offer opportunities for the diagnosis of difficulties and immediate intervention, thus helping students who are struggling with important mathematics content.

What characteristics qualify a concept or topic to be a curriculum focal point? For inclusion in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics*, a focal point had to pass three rigorous tests:

- Is it mathematically important, both for further study in mathematics and for use in applications in and outside of school?
- Does it “fit” with what is known about learning mathematics?
- Does it connect logically with the mathematics in earlier and later grade levels?

A curriculum focal point may draw on several connected mathematical content topics described in *Principles and Standards for School Mathematics* (NCTM 2000). It should be addressed by students in the context of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation. Without facility with these critical processes, a student’s mathematical knowledge is likely to be fragile and limited in its usefulness.

A complete set of curriculum focal points, situated within the processes of mathematics, can provide an outline of an integrated mathematics curriculum that is different from the outline created by a set of grade-level mastery objectives or a list of separated content and process targets. In contrast with grade-level mastery objectives, which can be interpreted as endpoints for learning, curriculum focal points are clearly areas of emphasis, calling for instruction that will help students learn content that gives them a foundation for increasing their understanding as they encounter richer and more challenging mathematics.



Instruction based on focal points would devote the vast majority of attention to the content identified for special emphasis in a grade. A curriculum for pre-K–8 based on a connected set of such focal points could provide a solid mathematical foundation for high school mathematics.

3

How Should Curriculum Focal Points Be Used?

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics highlights important mathematics at particular grade levels for the use of those who are responsible for the development of mathematics curricula, standards, and assessment—primarily the mathematics education leaders and policymakers at the national, state, and local levels. As a result of the wide variation in the placement of topics in current mathematics curricula (Reys et al. 2006), the grade-level designations of particular curriculum focal points in this publication may not match the placement of the corresponding content in an existing curriculum. This publication is presented as a framework on which the next generation of state and district-level mathematics curricula might be built. Organizational strategies that embody the field’s best thinking, such as these focal points, can serve as a catalyst to curriculum development and positively influence the design of materials for instruction and assessment.

The set of curriculum focal points described here represents an attempt to provide curriculum developers with a clear organizational model for establishing a mathematics curriculum from prekindergarten through grade 8 by identifying for each grade level important content that can build connected and integrated mathematical understanding. The curriculum focal points and their accompanying “connections” to related content outline instructional targets for a basic, integrated, grade-by-grade framework for a coherent mathematics curriculum.

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics does not specify instructional approaches for the implementation of the suggested curriculum focal points. Its presentations of the focal points include neither suggestions for tools to use in teaching nor recommendations for professional development in content or pedagogy. The focal points cannot be used alone as lesson plans. Nor do they answer the question, “What should I do in class on Monday?” Nevertheless, the curriculum focal points identified here should be of considerable interest to teachers and other practitioners, as well as curriculum developers and policymakers.

To achieve the best results with students when teaching for the depth, understanding, and proficiency sought by the curriculum focal points, teachers themselves will need a deep understanding of the mathematics and facility with the relationships among mathematical ideas. Thus, effective instruction built on the curriculum focal points requires in-depth preparation of preservice teachers and ongoing professional development for in-service teachers.

4

How Do the Curriculum Focal Points Relate to Principles and Standards for School Mathematics?


Principles and Standards for School Mathematics (NCTM 2000) describes the foundational mathematical ideas on which the focal points in *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* rest and toward which they direct students' learning. *Principles and Standards* remains the definitive reference on the development of mathematical content and processes across the grades. Since the publication of this influential work in 2000, ideas like *coherence*, *focus*, *high expectations*, *computational fluency*, *representation*, and *important mathematics* have become regular elements in discussions about improving school mathematics, and thinking about these ideas has evolved considerably. As the next step in devising resources to support the development of a coherent curriculum, NCTM now offers a new publication, with a set of curriculum focal points and connections for mathematics education in prekindergarten through grade 8.

Principles and Standards includes a thorough discussion of the necessity for learning mathematical content through the processes of problem solving, reasoning and proof, communication, connections, and representation. Although some of these processes may be evident in the descriptions of particular focal points, this new publication primarily targets *content*. Its presentation of curriculum focal points assumes that the mathematical processes described in *Principles and Standards* will be implemented in instruction that requires students to discuss and validate their mathematical thinking; create and analyze a variety of representations that illuminate the connections within the mathematics; and apply the mathematics that they are learning in solving problems, judging claims, and making decisions.

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics identifies three focal points at each grade level. Each set of three focal points, together with the integrated content taught in the context of the processes, should encompass the major portion of instruction at that grade level. The presentations of the focal points for each grade level also identify “Connections to the Focal Points” in a column at the right. These connections serve two purposes:

1. They recognize the need for introductory and continuing experiences related to focal points identified for other grade levels.
2. They identify ways in which a grade level's focal points can support learning in relation to strands that are not focal points at that grade level.

The “Connections to the Focal Points” column for each grade level brings in other important topics in meaningful ways. For example, the grade 2 “Connections” highlight the fact that the Measurement Focal Point for grade 2 (“Developing an understanding of linear measurement and facility in measuring lengths”) includes work with applications and models using the shapes from the Geometry Focal Point for grade 1 (“Composing and decomposing geometric shapes”). At the same time, students in grade 2 continue to use vocabulary and spatial reasoning that will be essential for learning the content specified in the Geometry Focal Point for grade 3 (“Describing and analyzing properties of two-dimensional shapes”). Because a curriculum that is integrated



and internally connected in this way uses related concepts and skills to support and enrich one or more focal points at a grade level, it has the potential to maximize students' learning.

Each focal point in this publication takes its name from the content strand or strands to which it relates in *Principles and Standards for School Mathematics*. Many focal points relate to more than one content strand, highlighting the integrated nature of the curriculum focal points. That single focal points are often described with a combination of items from different content strands in *Principles and Standards* reflects the fact that *Principles and Standards* itself presents “a connected body of mathematical understandings and competencies ... rather than a menu from which to make curricular choices” (p. 29). Color-coded comparison charts in the appendix illustrate the extent to which the curriculum focal points and their connections include content that *Principles and Standards* expects instruction to address in the corresponding grade bands.